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End Use Elasticities

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END USE ELASTICITIES

by Joseph G. Hirschberg

1 Introduction

This paper demonstrates a method for estimating relative price elasticities which employs the second moments of the vector of the quantity demanded. This method is based on the observation that observed randomness in demand is due to variation in either preferences, prices, or income. In the short-run, (in this case during a month) we assume that preferences and income remain constant. Consequently, observed demand variation is due to perceived price changes even when no monetary price changes occur, by assuming that the perceived price includes a shadow component. By making assumptions concerning the stochastic process of the shadow prices and the form of the demand relationship we can identify relative price elasticity estimates from usage data with no observed monetary price variation.

Models which use the second moments of demand relationships to estimate elasticity and substitution relationships have been proposed by a number of authors. Theil and Neudecker (1958) first propose the examination of the residuals from Engel curves though they do not attempt to apply the concept. Phlips (1971) and Phlips and Rouzier (1972) provide an empirical example based on the application of a Houthakker-Taylor demand model in which the residual cross-equation covariance matrix from a demand system is examined for signs of misspecification. Both Theil (1971) and Phlips (1974, page 209) propose models in which the covariance of demand is made a function of the preference structure via a stochastic component in the linear portion of a quadratic utility function. The present model differs from these models in that it assumes that stochastic process in the observed usage is generated by shocks in the prices.

We estimate the relative price elasticities by month and household for a panel of households whose electricity consumption was monitored by time-of-day (TOD) but were not placed on rates that varied by TOD. These households constituted the control group in an experiment to measure the impact of TOD prices on the demand for electricity that was conducted by the Los Angeles Department of Water and Light. The elasticities are then made the dependent variable in a second stage analysis employing a regression with the independent variables defined as dummy variables which represent presence of a particular electric appliance.

This analysis can be viewed as a model in another dimension from the extensive literature that has formed in the area of conditional demand analysis as applied to forecast end-use demand for energy. The first work in this area was by Parti and Parti (1980) with more recent extensions employing additional data and more sophisticated econometric models in Aigner, Sorooshian and Kerwin (1984), Caves, Herriges, Train, and Windle (1987), Bartels and Fiebig (1990), Hsiao, Mountain and Ho (1990), Fiebig, Bartels and Aigner (1991), Fiebig, and Bartels (1991) and Bauwens, Fiebig, and Steel (1994). The technique employed here could well be improved upon with more sophisticated econometric methods and the availability of appliance specific usage data as has been done with the models of demand.

2 A Logarithmic Model

This section is a summary of the description of the model that appears in Hirschberg (1994). Let us assume that TOD prices include a stochastic component. Thus the log of the perceived price (P) is defined as:

$$P = P_o + (P_s + \varepsilon) \tag{2.1}$$

where P_s = log of the shadow price and it is assumed that the shadow price is equal to one for every hour thus $P_s = 0$ for every hour, P_o = log of the observed monetary price (as defined by existing TOD rates) and ε = a random error in the perception of the log shadow price where $E[\epsilon_t] = 0$, $E[\epsilon_t^2] = \sigma^2$ and $E[\epsilon_t \epsilon_s] = 0$ when $s \neq t$.

A simple logarithmic demand relationship for 24 hourly commodities (or services):

$$X_{(nx24)} = P_{(nx24)} \mathrm{E}_{(24x24)} \tag{2.2}$$

where n = the number of days in the sample, X = log of the hourly demand for services, P = log of the perceived price by time of day, and E = matrix of price elasticities. From the elasticity version of the Slutsky equation we can obtain the relationship;

$$e_{ji}w_j + e_{jm}w_i w_j = e_{ij}w_i + e_{im}w_i w_j, \tag{2.3}$$

where e_{ji} is the Marshallian or uncompensated price elasticity of commodity j with respect to price i, e_{jm} is the income elasticity of commodity j, and w_j is the expenditure share of commodity j. Given the short time period of the data to be analyzed we assume that the income elasticities for the same good at different times of the day are equal to each other. Note also that in a situation where n=24 and no explicit TOD price differences the expenditure shares will average 1/24 and the $w_i w_j$ will be of the order of $(1/24)^2$ or .0017 thus $e_{ji}w_j - e_{ij}w_i \approx .0017(e_{im} - e_{jm})$ so that the differences in income elasticity need to be quite large to imply serious consequences from this assumption. From this assumption define $h_{ij} = e_{ij} w_i$ and $h_{ji} = e_{ji} w_j$ thus $h_{ij} = h_{ji}$ and we can define the n by n symmetric matrix **H** by

$$\mathbf{H} = \mathbf{E}\ \mathrm{diag}(w) \tag{2.4}$$

where diag(w) is an n by n matrix. **H** can also be written as a function of the Slutsky matrix

$$\mathbf{H} = \mathrm{diag}(\mathbf{p})\ \mathbf{S}\ \mathrm{diag}(\mathbf{p})\ m - \mathrm{diag}(w)\ e_m\ \mathrm{diag}(w), \tag{2.5}$$

where diag(**p**) is a matrix with the prices on the diagonal, m is the income and e_m is a matrix with the income elasticities for commodity i in every column of row i. Under the assumption that all $e_{im} = e_{jm}$, e_m is a matrix with equal elements, thus **H** is symmetric. From this relationship it can be shown that a necessary condition for **S** to be negative semidefinite is for **H** to be negative semidefinite as well.

Using this model, and observations of hourly service demand for a number of days over which we assume that P_o, $E(P_s)$, and **E** remain constant we can write the expected value of the level of usage as

$$E(X) = (P_o + P_s)\ \mathbf{E}, \tag{2.6}$$

and obtain

$$X - E(X) = \varepsilon\ \mathbf{E}. \tag{2.7}$$

(2.4) can be solved for **E**,

$$\mathbf{E} = \mathbf{H}\ \mathrm{diag}(w)^{-1}. \tag{2.8}$$

The covariance of the observed X is given by:

$$\mathrm{cov}(X) = \sigma^2 \mathrm{diag}(w)^{-1} \mathbf{H}^T \mathbf{H}\ \mathrm{diag}(w)^{-1}. \tag{2.9}$$

To estimate **H** we can employ the eigenvalue decomposition of a matrix formed by pre and post multiplication of the covariance matrix by diag(w).

$$\mathrm{diag}(w)\ \mathrm{cov}(X)\ \mathrm{diag}(w) = \mathbf{L}\ \Lambda\ \mathbf{L}^T, \tag{2.10}$$

where a symmetric and negative semidefinite estimate of $\sigma^2\mathbf{H}$ is given by

$$\hat{\sigma}\hat{\mathbf{H}} = \mathbf{L}\ \ -\Lambda^{1/2}\ \mathbf{L}^{T}. \qquad (2.11)$$

thus, $\mathbf{L}$ = matrix of eigenvectors for cov(X) as each column, and $\Lambda^{1/2}$ = diagonal matrix of the square roots of the eigenvalues for cov(X). It can be shown that $\hat{\sigma}\hat{\mathbf{H}}$ is a unique solution (see Theil and Neudecker 1958). We can define $\hat{\sigma}\hat{\mathbf{E}}$ by

$$\hat{\sigma}\hat{\mathbf{E}} = \hat{\sigma}\hat{\mathbf{H}}\ \mathrm{diag}(\mathrm{w})^{-1} \qquad (2.12)$$

Note that we can only identify **E** up to a scalar multiple and, in the application given below, we only refer to relative elasticities, and not to absolute values.

While it may seem odd that we can identify **E** without reference to the monetary prices, if it is the case that consumers react to opportunity costs and if we assume these shadow prices are driven by stochastic processes, as shown above, the identifying assumption is consistent with observed variations in demand levels. In effect, the stochastic prices are translated into variations in demand via the elasticities defined in this model as **E**.

3 Time-of-Day Electricity Demand

The application of this model was made to TOD electricity demand data. The data set used consists of household level observations by day for each hour. Along with the electricity demand data, we also have the results of a survey of each household in which demographic and appliance stock data was obtained. This data set was collected as part of the study conducted to investigate the impact of TOD electricity rates although the customer's data sets selected for this analysis are from the control group, consequently, they were not subjected to TOD energy rates. (See Hirschberg 1989 for a more complete description of this data.) In the following analysis we estimate the hour by hour elasticities by use of the model described above. Then in a second step we determine the impact of the ownership of appliance stocks to influence the hourly elasticities.

The electricity usage data were collected over a three year period and were selected for the winter months when cooling demand will have a smaller chance to dominate the components of electricity demand. There are 145 households in the control rate and seasonal groups (LADWP defined these rates as 101, 102, 201, 202, 211, and 212). The data are the integrated hourly demand for electricity in kW recorded at the end of each hour using tape recording devices.

Before estimating the elasticities it was first necessary to remove the systematic factors that impact the daily data. The presence of calendar effects and secular trends will influence our results if we do not account for these non-random changes in daily usage patterns. To this end, we first compute a set of regressions on the log of the electricity demand as a function of dummy variables for days of the week and a cubic in time.

The equation for the log of a particular hour's demand for a household-month can be written as:

$$x_{i_{(nx1)}} = (P_o + P_s + \epsilon)_{(nx24)} E_{i_{(24x1)}} + Q_{(nxk)} D_{i_{(kx1)}} + u_{i_{(nx1)}} \qquad (3.1)$$

or

$$x_i = (P_o + P_s) \mathbf{E}_i + \mathbf{Q} \mathbf{D}_i + \varepsilon \mathbf{E}_i + u_i, \qquad (3.2)$$

where **Q** is the matrix of dummy variables for each day of the week and a cubic in time, Di is the vector of coefficients that corresponds to the columns of **Q** and $\mathbf{E}_i$ is the ith column of **E**. To facilitate estimation we assume $u_i = 0$ so that all error in the model is contained in the $\varepsilon \mathbf{E}_i$ term. The relaxation of this assumption may be possible when special assumptions are made concerning the form of E, i.e. that the elasticities for all hours from 1 a.m. to 4 a.m. are the same. The regression is of the form :

$$xi = \alpha_i + \mathbf{Q}_i \mathbf{D}_i + e_i, \tag{3.3}$$

where $\alpha_i = (P_o + P_s) \mathbf{E}_i$ and $e_i = \varepsilon \mathbf{E}_i$. The residuals from (3.3) denote the variance in demand that is not explained by a systematic model. From (2.9) we can show the relationship between the covariance of e_i and H as

$$\text{diag(w) cov}(\hat{e}) \text{ diag(w)} = \sigma^2 \mathbf{H}^T\mathbf{H}. \tag{3.4}$$

where $\hat{e}$ is the (24xn) matrix of residuals from the set of 24 regressions fit to (3.3). Using the solution procedure as defined above we can then define $\hat{\sigma}^2\hat{\mathbf{E}}$ as

$$\hat{\sigma}^2\hat{\mathbf{E}} = \mathbf{L}\ \text{-}\Lambda^{1/2}\ \mathbf{L}^T\ \text{diag(w)}^{-1}. \tag{3.5}$$

where L and Λ are the eigenvectors and eigenvalues of the matrix on the left side of (3.4).

The estimate of $\hat{\sigma}^2\hat{\mathbf{E}}$ obtained above is a point estimate. Furthermore, we use the relative elasticities in reporting these results. An element of $\hat{\sigma}^2\hat{\mathbf{E}}$ is chosen as the divisor (in the present case the first row, first column). The relative elasticity matrix (RE) is defined as

$$\hat{\mathbf{R}}\hat{\mathbf{E}} = \frac{\hat{\sigma}^2\hat{\mathbf{E}}}{\hat{\sigma}^2\hat{\mathbf{E}}_{1,1}}. \tag{3.6}$$

In order to obtain variance estimates of this ratio of square roots of a modified covariance matrices we use Efron's (1982) bootstrap. Beran and Srivastava (1985) demonstrate the use of the bootstrap method for functions of the covariance matrix. Note that one of the advantages of this model is the low computational expense for the calculation of the point estimates of $\hat{\sigma}^2\hat{\mathbf{E}}$, which allows the inexpensive application of the bootstrap technique. 20 bootstrap resamplings were used from the residuals in (3.3) for each month-household to estimate the variances of the relative elasticities.

We estimate $\hat{\mathbf{R}}\hat{\mathbf{E}}$ and its variance for each household and month (1064 cases). The diagonal elements of $\hat{\mathbf{R}}\hat{\mathbf{E}}$ are then used as the independent values and in a regression model of the following form;

$$\hat{\mathbf{R}}\hat{\mathbf{E}}_{ii} = \alpha_i + \sum_{j=1}^{14} \beta_{ij} Z_j + \zeta_i, \; i = 2 \text{ to } 24. \qquad (3.7)$$

where $\hat{\mathbf{R}}\hat{\mathbf{E}}_{ii}$ is the ith hour relative own-price elasticity, α, β, ϕ, and γ are parameters and Z_j are independent variables. The appliances for which we have information are: air conditioners (AC=1), dish washer (DWASH=1), electric; clothes dryer (EDRY=1), cooking range (ERANGE=1), space heating (ELHEAT=1) and water heater (EWHEAT=1). The demographic variables are; income (INC), number of persons in the household (NHH), proportion of the household that is less than 21 (PLT21), and the proportion of the persons in the household over 65 (PGT65). The building characteristics are defined as; the number of rooms (NR) and whether the house is attached to another structure (HATT=1). The weather variables used are the average cooling (CD) and heating degree days for the month (HD) (see Hirschberg 1989 for additional details concerning the definition of these values). It is assumed that the errors (ζ_i) are heteroscedastic, with a variance proportional to the variance of the relative elasticities. Accordingly, each equation is estimated by employing a weighted least squares procedure where the inverse of the bootstrap estimated standard error for each relative own-price elasticity was used as the weight.

As mentioned in the introduction, this model is similar to the type of model used in conditional demand analysis for forecasting end-use electricity usage. Except for the introduction of the heteroscedastic error structure it is most similar to early work by Parti and Parti 1980 and Aigner, Sorooshian and Kerwin 1984) in which they use traditional regressions to decompose total household energy demand by household appliance stock.

4 Results

Table 1 lists the coefficient estimates from estimating equation (3.7) (the coefficients multiplied by 100 above the t-statistics). We can see from Table 1 that over 55% of the 322 non-intercept parameters estimated in the 23 equations of the form of 3.6 have estimated standard errors rejecting the hypothesis that the coefficient is equal to zero at the 95% or greater level of confidence. And for a number of cases (AC, DWASH, EDRY, ELHEAT, ERANGE, INC, NHH, PGT65 and PLT21) we can reject the null hypothesis of no effect in well over half of the equations.

One method to view these results is in the form of plots by time of day as given in Figures 1-15. In these figures the horizontal axis is the time of the relative own-price elasticity and the vertical axis is the estimated coefficient on the variable for that particular characteristic. In Figure 1 we have the intercept term which indicates that proportion of the relative own-price elasticity that is unexplained by the variables in the model. The lines in these plots provide the smoothed value of the parameters along with the upper and lower 95%

confidence interval based on the estimated standard errors for each coefficient, the plotted points are the coefficient values. The smoothing used tends to enlarge the confidence bounds, so that in some figures coefficients that are statistically significant from zero at the 95% confidence level appear not to be. Note from Figure 1 that the largest proportion of unexplained variation occurs in the elasticities for the period from 8 a.m. to 11 p.m.. Recall that these elasticities are relative to the own-price elasticity from midnight to 1 a.m., so we expect that the elasticities during day-time will be more elastic than those in the early morning and late at night.

From Figure 2 it appears that having an air conditioner implies a lower relative elasticity in the late afternoon hours when the demand for cooling is greatest. Thus households with air conditioning will be less flexible in their demand for electricity than those without, particularly in the afternoon.

In Figure 3 we can see the impact of the presence of a dishwasher on the relative own-price elasticity for electricity is greatest in the afternoon which indicates that use of a dishwasher in the afternoon is more variable than its use in the morning before noon.

The influence of the use of an electric clothes dryer is provided in Figure 4. The mid-morning and late afternoon appear to be the periods of lowest relative own-price elasticity.

Electric heaters (as can be seen in Table 1 and Figure 5) exert a greater influence than air conditioners. This due to choice of winter months for the data. The higher relative elasticity for electrically heated houses reflects the wide fluctuation in these households demand even when conditioned by the weather. The electric heating requires such a large proportion of the total electricity demand that it dominates the elasticity values.

The plot of the coefficients for the presence of an electric range (see Figure 6) demonstrate the nature of the demand for cooking. In the late morning the demand exhibits the greatest variation with the greatest elasticity around the 5 p.m. to 7 p.m. period.

The impact of electric water heaters on the relative own-price elasticity (see Figure 7) shows a relationship in which the variation is the least after the dinner period at 6 p.m. to 8 p.m.

In Figure 8 we view the variability due to whether a house is attached or unattached. The implication of being attached is that the house is an apartment. Less than half of the coefficients for apartments are significantly different from zero and for those that are significant occur during the night-time. This may indicate that HATT is a proxy for better insulation.

Income appears to have a significant negative impact on elasticity for those hours from 9 a.m. to 11 p.m. with a major impact at 6 p.m. (see Figure 9). This indicates that as income rises the elasticity for these hours of the day becomes less and less. This finding could be used to assess the distributional effects of a proposed TOD rate.

From Figure 10 it can be seen that as the number of occupants in the household increases the impact on the relative own-price elasticities appears to be positive in the early morning and in the early afternoon and significantly negative from 7 to 9 p.m.. The impact of this variable should be considered in conjunction with the effect of income.

The number of rooms (Figure 11) has a small impact on the relative own-price elasticities. Only six of the hourly parameters are significantly different from zero. The highest values are recorded for the 5 p.m. and 6 p.m. time periods and these may indicate that the larger the number of rooms the greater the variation in cooking at home -- this could be interpreted as an other indication of income.

The plot of the coefficient for the proportion of the household greater than 65 years of age (Figure 12) shows the possibility that these occupants are not home during the period from 7 a.m. to 5 p.m. indicating that this period has a high degree of variability in the demand and thus in the elasticity.

Figure 13 gives the relative elasticity as a function of the proportion of the household that is under 21 years of age. This plot appears to be the direct opposite image of Figure 12 which may indicate the differing at-home patterns of these households.

The last two Figures (14 and 15) show the impact of weather on relative price elasticities. These results indicate that greater weather demands force produce higher relative own-price elasticities in the late afternoon. It is interesting to note that the heating degree parameters are measured with considerably more error than the cooling degree ones. This may indicate a wider latitude in the use of heating devices than in the use of cooling devices.

5 Conclusion

The model proposed and estimated in this paper provides a method for the interpretation of the variation observed in the demand for a service (here TOD electricity) as an indication of the elasticity of demand for that service. The primary result of this application is the ability to predict the set of relative price elasticities for any type of customer based on a model in which each element of the estimated relative elasticity matrix $\hat{\mathbf{R}}\hat{\mathbf{E}}$ can be defined as a function of the household characteristics and the weather. Here we have limited the scope of analysis to the diagonal elements of $\hat{\mathbf{R}}\hat{\mathbf{E}}$, however the entire relative cross price substitution matrix can be considered with the same model specified in (3.7). In addition, it has been shown that a bootstrap can be used to compute the variance of each household's elasticity estimate so that the secondary regression analysis can include a weight for each observation that accounts for the quality of each household and month level estimate. Future uses for this model include the computation of welfare measures for various scenario TOD rates and household characteristics.

The simplicity of this model and the high level of detail in the results, makes this type of analysis a convenient "first step" in the consideration of any pricing proposal which involves the differentiation of a service which was has not previously been subject to pricing differentials. A preliminary welfare analysis of the impact of any proposed rate can be made using the aggregated relative elasticity matrix or dissaggregated through the ability to taylor the relative price elasticities by household type. The detailed data used in the analysis could also be aggregate data -- hourly observations which are summed over individual demand. Another extension may be to the case were some price differentiation exists, but it may be of interest to investigate more detailed patterns of substitution than those available from traditional regression analysis of the first moments.

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FIGURES 1 - 15

and

TABLE 1

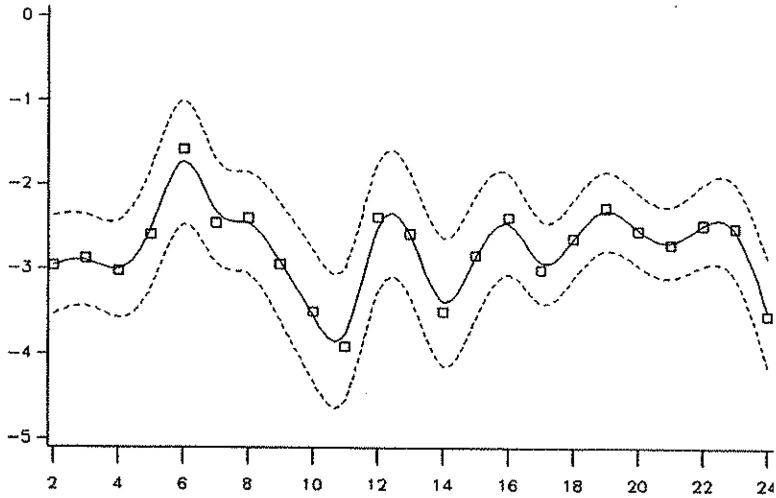

Figure 1 Intercept (INTERCEP)

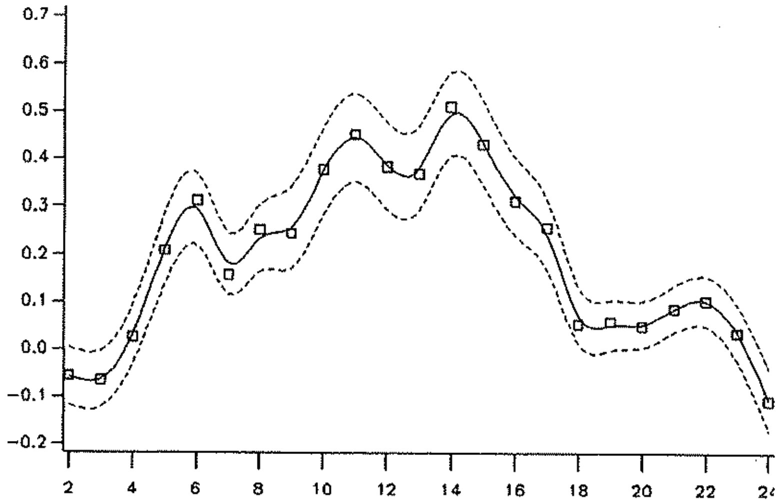

Figure 2 Air Conditioning (AC)

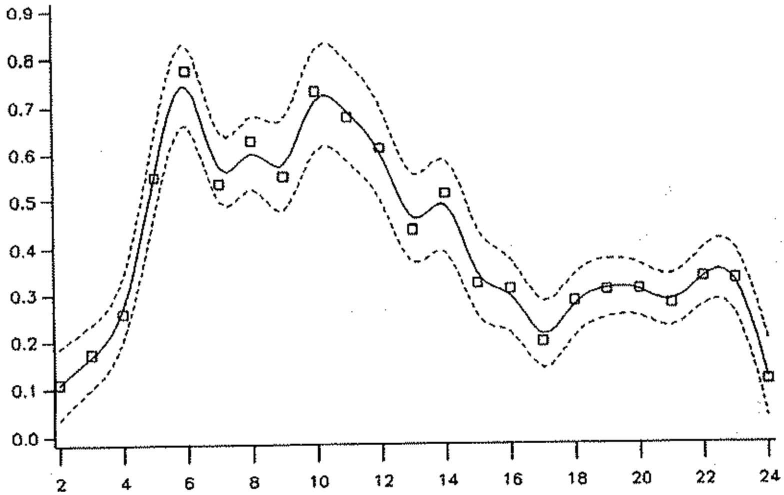

Figure 3 Dish Washer (DWASH)

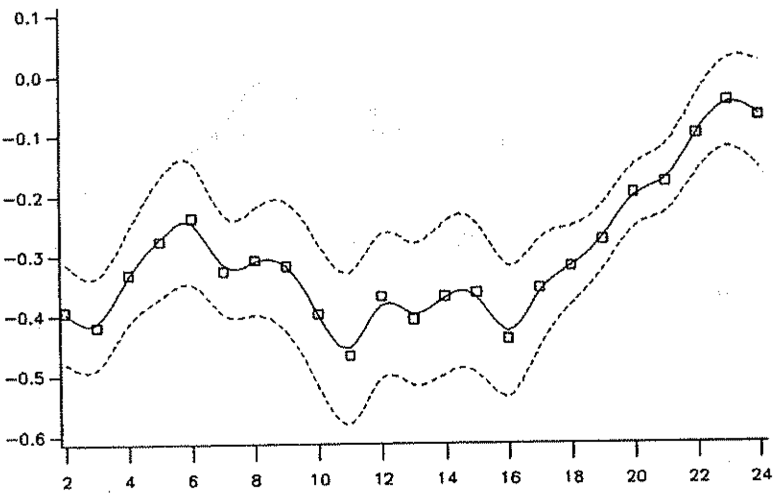

Figure 4 Clothes Dryer (EDRY)

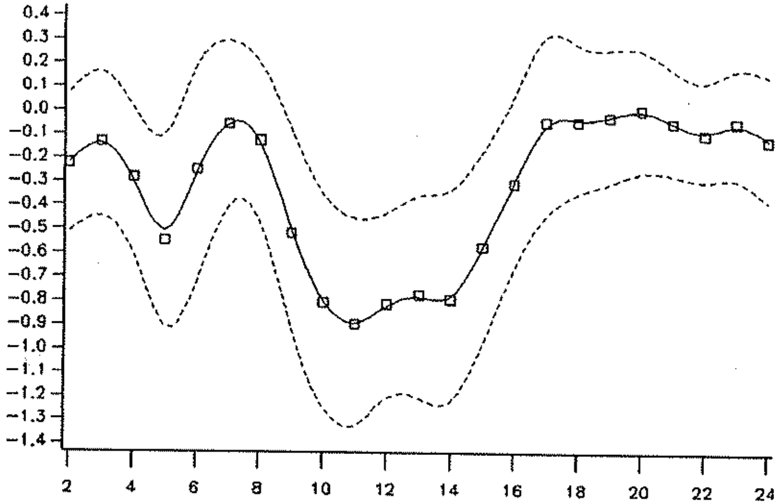

Figure 5 Space Heater (ELHEAT)

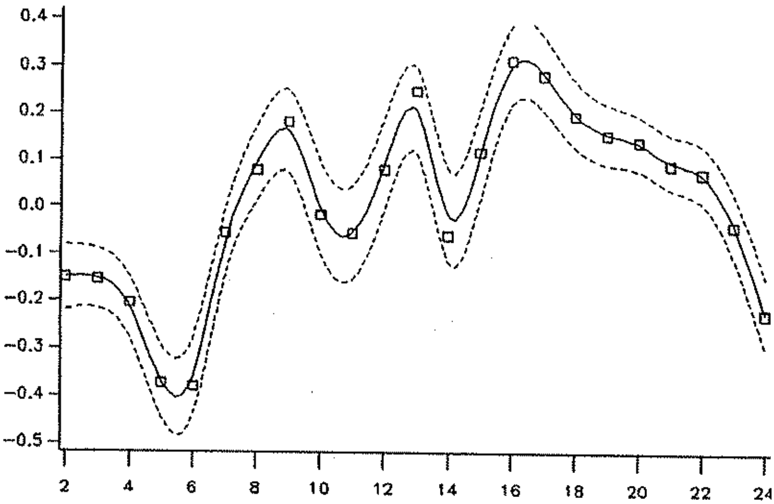

Figure 6 Cooking Range (ERANGE)

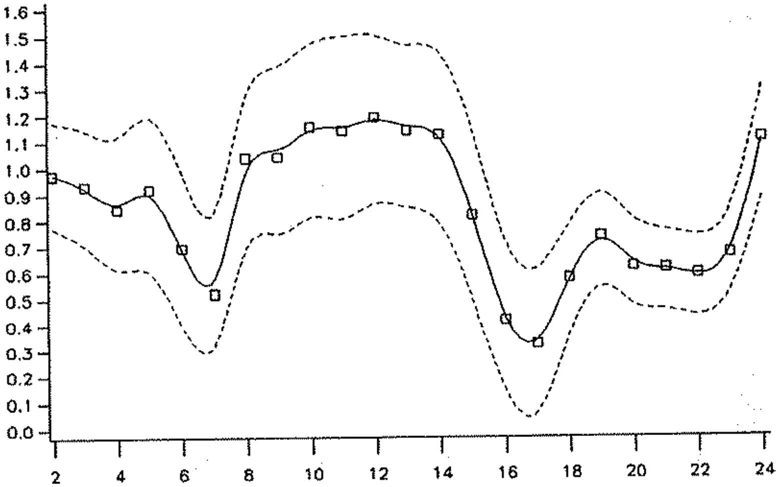

Figure 7 Water Heater (EWHEAT)

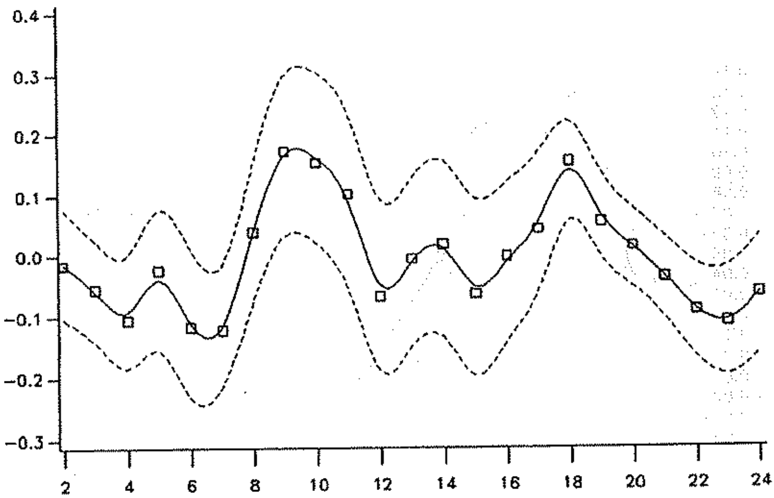

Figure 8 Attached House (HATT)

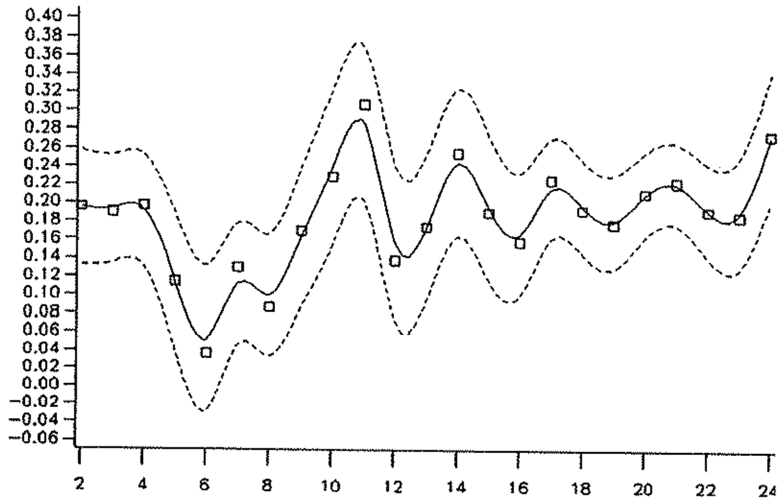

Figure 9 Income (INC)

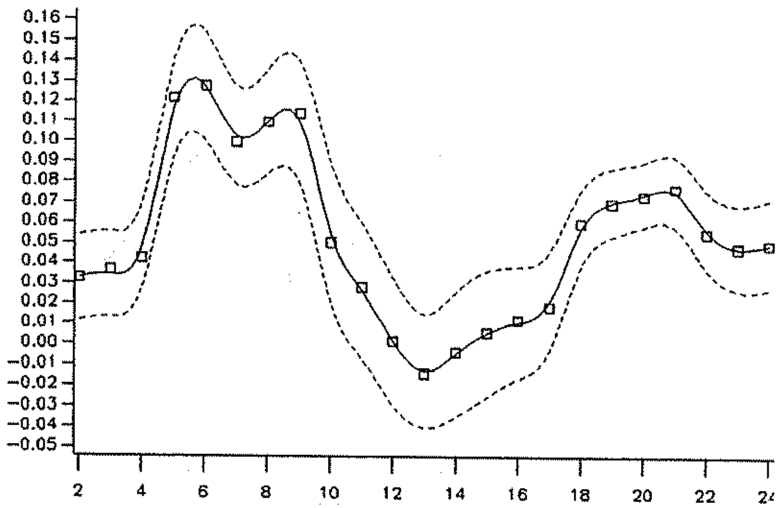

Figure 10 Persons in Household (NHH)

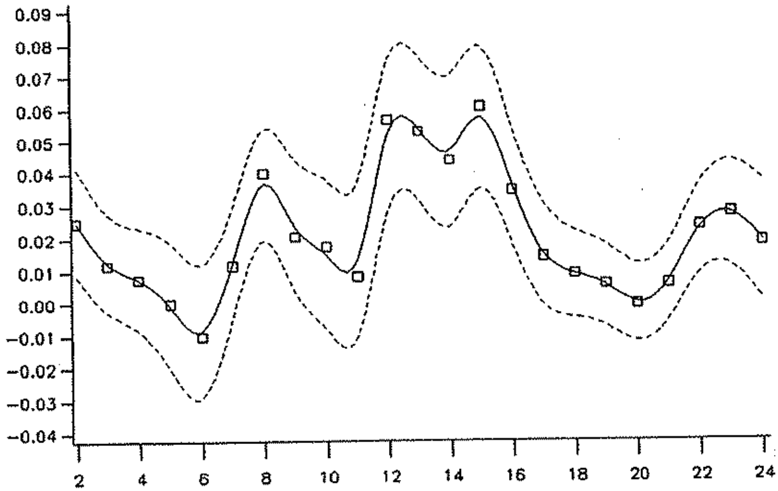

Figure 11 Rooms in House (NR)

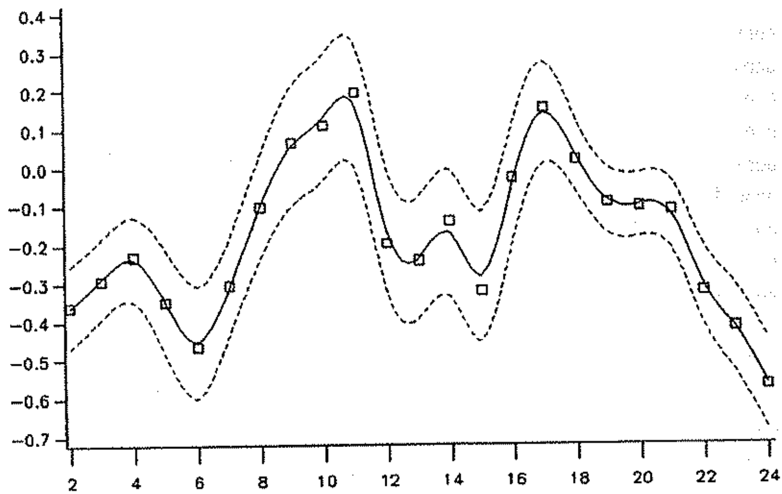

Figure 12 × of Household > 65 yrs (PGT65)

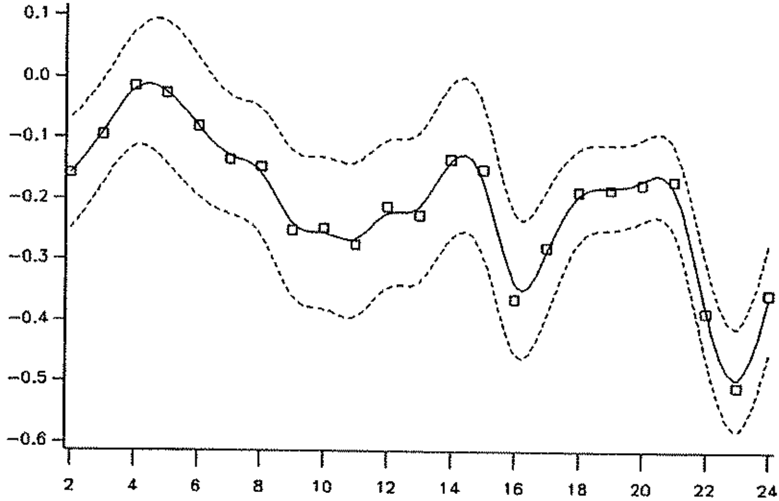

Figure 13 % of Household < 21 (PLT21

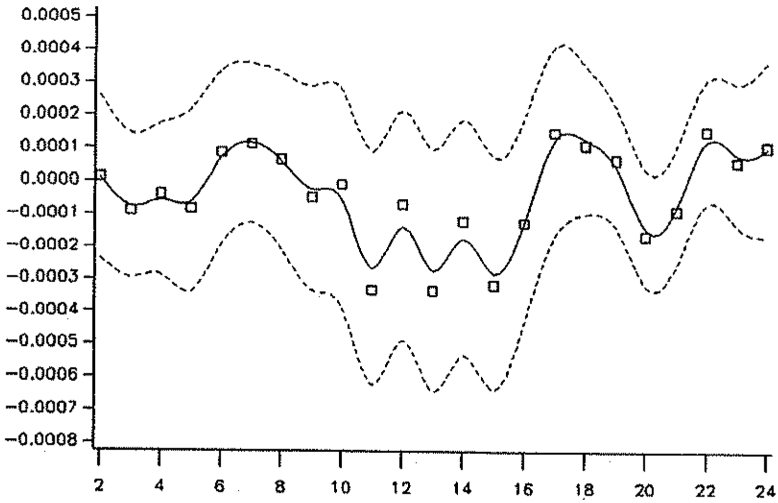

Figure 14 Cooling Degree Days (CD)

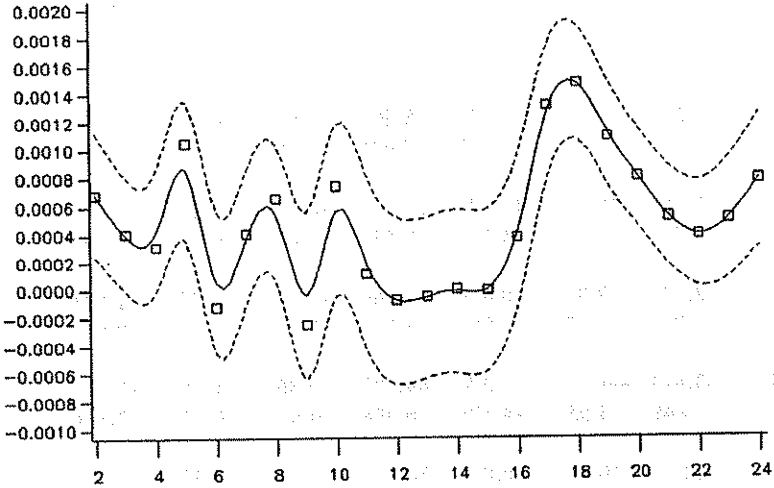

Figure 15 Heating Degree Days (HD)

Table 1 - Coefficients (x 100) over t-statistics

Time	Intercept	AC	Dwash	Dryer	Heater	Range	Wheater	Att
2	5.767	-1.520	-8.570	7.535	48.407	-6.032	-10.05	-11.36
	0.287	-0.675	-3.162	2.599	7.626	-2.459	-1.899	-3.415
3	-35.36	11.560	-10.35	11.928	66.595	-6.392	4.805	-15.84
	-1.846	5.785	-4.315	4.535	8.582	-2.679	0.738	-5.493
4	-38.98	2.074	-7.301	-0.410	61.669	-0.002	1.748	-1.583
	-2.212	1.081	-3.407	-0.206	8.037	-0.001	0.297	-0.629
5	-11.52	-3.613	-11.45	-7.376	49.737	0.862	-1.460	-7.155
	-0.623	-1.848	-5.554	-4.174	6.394	0.427	-0.236	-2.854
6	-15.74	-1.387	-10.73	10.079	38.729	5.615	-7.320	-10.77
	-0.742	-0.558	-3.715	3.716	4.001	2.226	-0.845	-3.236
7	17.390	8.381	-21.96	-0.933	54.318	-7.618	-11.82	-16.48
	0.595	2.363	-5.296	-0.263	4.530	-2.042	-1.014	-3.494
8	1.385	9.106	-14.67	-2.493	63.515	-3.016	30.159	-8.978
	0.047	2.154	-2.864	-0.475	5.137	-0.636	2.120	-1.377
9	162.48	15.107	0.040	-22.92	45.472	11.189	-13.43	-0.097
	4.558	3.217	0.007	-4.041	3.177	2.219	-0.922	-0.014
10	161.49	6.155	7.324	-22.59	49.635	17.890	2.555	10.564
	4.837	1.225	1.352	-3.420	3.287	3.385	0.163	1.346
11	146.75	10.794	-19.39	-28.13	64.338	21.904	-16.19	3.339
	4.915	2.376	-3.732	-4.455	5.070	3.970	-1.227	0.463
12	151.21	-0.859	-7.794	-22.74	50.342	14.739	-20.11	-6.283
	4.233	-0.174	-1.392	-3.318	4.192	2.871	-1.598	-0.811
13	150.74	-2.981	-10.23	-10.18	54.753	24.109	-39.44	-12.26
	5.113	-0.650	-1.985	-1.638	4.451	4.754	-3.494	-1.743

Time	Income	# in HH	# of rooms	% > 65	% < 21	CD	HD
2	4.288	1.171	0.671	9.577	11.444	-0.020	0.006
	2.000	1.391	1.030	2.300	3.212	-2.330	0.378
3	6.240	-1.113	0.988	7.703	-0.101	-0.009	0.011
	3.002	-1.466	1.870	2.011	-0.032	-1.307	0.651
4	5.000	0.723	1.483	-6.153	-7.411	-0.003	0.046
	2.642	1.106	2.642	-2.122	-3.224	-0.564	3.554
5	4.406	-1.021	1.426	-9.975	-4.617	-0.014	-0.013
	2.193	-1.729	2.596	-3.948	-2.110	-3.877	-1.195
6	4.400	2.478	1.261	-14.77	-17.16	-0.010	-0.017
	1.850	3.008	1.749	-4.414	-5.712	-1.871	-1.279
7	0.190	5.755	1.545	-7.702	-6.271	0.006	0.034
	0.057	5.336	1.611	-1.741	-1.565	0.676	1.537
8	3.861	10.346	0.457	21.059	-16.00	-0.018	-0.052
	1.220	7.498	0.432	2.799	-2.888	-2.115	-1.652
9	-11.33	8.076	1.158	35.804	-29.50	-0.047	-0.054
	-2.990	5.169	1.028	4.104	-5.028	-3.472	-1.538
10	-10.82	8.155	1.022	40.673	-38.94	-0.017	-0.063
	-2.994	5.179	0.832	4.312	-6.277	-1.081	-1.810
11	-6.822	3.485	1.168	25.789	-26.46	-0.046	-0.049
	-2.225	2.421	0.913	3.041	-4.206	-3.245	-1.410
12	-7.128	3.277	0.329	38.191	-30.96	0.020	-0.024
	-1.889	2.344	0.271	4.201	-4.722	1.088	-0.727
13	-7.988	3.113	1.153	17.944	-28.67	0.021	-0.007
	-2.561	2.155	1.016	2.310	-4.657	1.234	-0.213

Table 1 continued - Coefficients (x 100) over t-statistics

Time	Intercept	AC	Dwash	Dryer	Heater	Range	Wheater	Att
14	104.47	-9.054	-18.25	-8.643	42.222	29.040	-21.27	-6.494
	3.631	-1.985	-3.565	-1.397	3.535	5.840	-1.910	-0.893
15	103.91	-11.52	0.656	-14.60	40.192	-0.595	-10.92	11.988
	3.263	-2.827	0.135	-2.411	3.823	-0.132	-1.108	1.656
16	107.66	-12.82	0.010	-23.15	19.266	9.480	-45.60	12.472
	3.522	-2.894	0.002	-3.787	1.693	1.770	-4.723	1.635
17	68.967	-15.52	12.848	-16.31	53.879	-11.05	-33.18	5.263
	1.964	-2.960	2.406	-2.390	3.924	-2.019	-3.014	0.632
18	144.45	-15.78	3.128	-32.48	33.597	17.855	-56.09	9.546
	3.913	-3.198	0.511	-5.235	2.769	2.895	-5.277	1.220
19	328.27	2.801	17.583	-26.14	29.180	6.972	-36.71	20.490
	8.627	0.670	3.018	-4.965	2.405	1.304	-3.880	3.067
20	195.14	-11.26	11.158	-17.29	9.276	3.641	-17.80	-7.257
	5.965	-2.824	2.188	-4.044	0.828	0.730	-2.100	-1.376
21	113.72	4.684	-4.041	-24.57	8.155	-25.16	3.413	-1.992
	3.355	1.280	-0.839	-5.601	0.787	-5.440	0.421	-0.389
22	148.77	-8.025	2.120	-20.2	-4.936	-13.9	5.377	3.173
	4.633	-2.251	0.461	-4.939	-0.520	-3.277	0.767	0.581
23	142.16	0.788	16.031	-23.98	-8.410	-6.782	-0.928	20.195
	3.903	0.221	3.664	-5.924	-0.899	-1.609	-0.128	3.679
24	17.676	12.846	20.541	-14.31	-42.29	-9.806	40.033	22.575
	0.588	4.096	5.396	-3.992	-4.799	-2.678	5.319	5.083

Time	Income	# in HH	# of rooms	% > 65	% < 21	CD	HD
14	-3.865	3.875	1.316	24.024	-12.82	-0.007	-0.022
	-1.317	2.703	1.079	3.123	-2.095	-0.451	-0.655
15	-7.337	8.318	0.934	17.584	-24.79	0.045	0.074
	-2.152	5.757	0.797	2.225	-4.245	2.679	2.458
16	-5.040	7.471	-2.682	-3.735	-13.97	0.048	0.106
	-1.539	4.347	-2.770	-0.425	-2.245	2.590	2.994
17	-2.311	5.582	-1.182	10.130	-1.399	0.080	0.066
	-0.619	3.202	-0.922	0.954	-0.177	3.509	1.810
18	-5.436	0.513	-2.164	-10.25	-4.468	0.033	0.078
	-1.388	0.287	-1.913	-1.022	-0.558	1.582	1.905
19	-25.59	-4.967	3.749	-51.89	7.974	0.018	-0.047
	-6.493	-2.926	3.197	-5.523	1.054	1.160	-1.310
20	-8.510	-5.089	-1.559	-45.78	41.937	0.009	-0.052
	-2.503	-3.501	-1.390	-6.140	6.206	0.545	-1.559
21	-0.849	-6.282	-0.862	-36.92	45.941	-0.000	-0.023
	-0.247	-4.006	-0.753	-4.801	6.852	-0.011	-0.721
22	-4.560	0.646	-1.047	-41.25	44.449	-0.025	-0.086
	-1.387	0.432	-0.996	-5.642	7.226	-1.788	-2.940
23	-5.595	7.279	-2.284	-38.46	19.336	-0.017	-0.103
	-1.493	4.772	-1.996	-5.445	3.455	-1.531	-3.560
24	7.103	-0.275	0.415	-40.87	20.992	0.003	-0.049
	2.232	-0.238	0.453	-7.537	4.612	0.220	-1.920

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