3rd International Workshop on Empirical Methods in Energy Economics (EMEE2010)

Surrey Energy Economics Centre (SEEC)
University of Surrey, UK
24th – 25th June 2010

NOTE:

The following Presentation represents *Work in Progress* for discussion at the EMEE2010 workshop. It therefore must not be referred to without the consent of the author(s).

Sponsored by:

Oil Prices: Heavy Tails, Mean Reversion and the Convenience Yield

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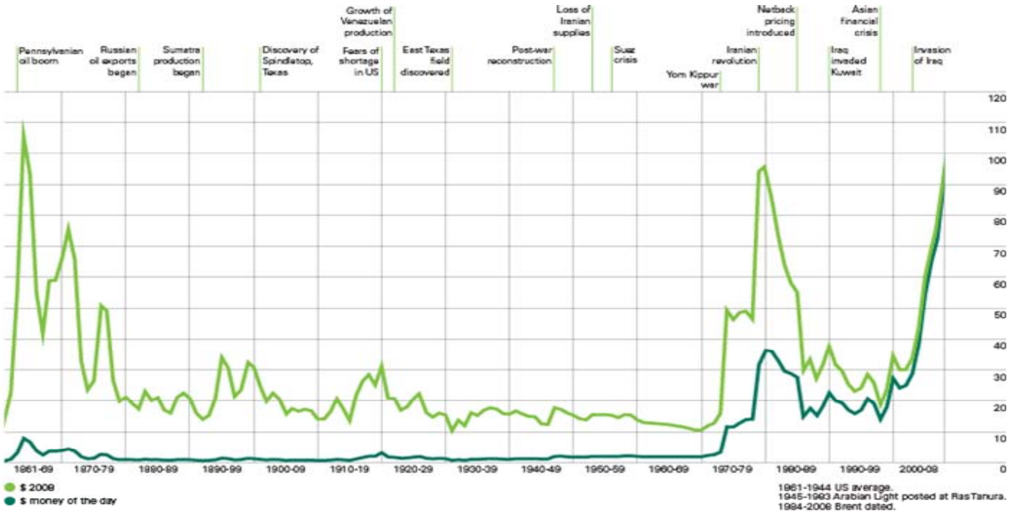
Chart of crude oil prices since 1861

Crude oil prices 1861 - 2008

US dollar per barrel
World events



MOTIVATION

- Analysis of oil price $\rightarrow$ active area of research:
  - linkages: financial markets or macro-economy [e.g. Kilian (2008a,b,c; 2009), Hamilton (2009)]
  - price dynamics, forecasting [e.g. Lee, List & Strazicich (2006), Cortazar & Naranjo (2006), Tabak & Cajueiro (2007), Postali & Picchetti (2006)].
- Stylized facts: heavy tails & breaks $\oplus$ statistical support for many models.
- Methodological disagreements:
  - deterministic/stochastic trends?
  - structural change / inherent fluctuations
  - unexpected discontinuities
  - non-constancy of variance
  - non-constancy of convenience yield.

RESEARCH QUESTION

- We examine mean-reversion in oil price from forecast-based perspective.
- We consider alternative classes of models with non-constancies in level or volatility.
- We do not take a stand on worth of unit root tests for the problem at hand, but we do believe that available evidence on the importance of parameter non-constancies, whether for refuting or for justifying the unit-root hypothesis, should be taken seriously.

- Theoretical reasons: unit-root not appropriate for natural resources or commodities:
  - Demand & Supply: when prices are higher (or lower) than some equilibrium level, high-cost producers will enter (or exit) the market, which pushes prices downward (or upward).
  - Relationship between (information in) future prices at different maturities & spot price ▶ convenience yield [**CY**].

- CY: flow of goods & services that accrues to owner of a spot commodity (**a physical inventory**) but not to owner of a futures contract (**a contract for future delivery**).
  - Random-walk [**RW**]: consistent with constant CY.
  - Mean Reversion [**MR**] & positive correlation between spot price & CY changes: consistent with theory of storage: when inventories $\downarrow$ (or $\uparrow$), spot price will $\uparrow$ (or $\downarrow$) & CY will also $\uparrow$ (or $\downarrow$) because futures prices will not $\uparrow$ (or $\downarrow$) as much as spot prices.
- [Pindyck (1999, 2001), Schwartz (1997), Schwartz & Smith (2000)]: refute constant CY & suggest MR to long-run equilibrium that itself can change randomly over time.

$\boxed{\text{CONTRIBUTION}}$

- We analyze a MR class of models from Schwartz (1997), Schwartz & Smith (2000), relative to various RW alternatives, with focus on forecast performance.
- The MR structural form:
  - presumes a stochastic CY,
  - derives from joint behavior of spot & future prices
  - allows one to disentangle the persistent (or long-run [**LR**) from the transitory (or short-run [**SR**]) component of price

$$\mathbf{LR} \blacktriangleright \text{Brownian motion;}$$
$$\text{SR} \blacktriangleright \text{Ornstein} - \text{Uhlenbeck}$$

$\Longrightarrow$overall evolution $\neq$ standard RW walk via ST deviation term.

- Non-MR class of models considers:
  - various (G)ARCH effects including asymmetric, conditionally non-normal (G)ARCH and (G)ARCH-in-mean [see Regnier (2007), Beck (2001), Sadorsky (2006)
  - random jumps: an integral part of price process leading to relatively rare adjustments that can be distinguished from frequent and relatively "small" ordinary price fluctuations. Ait-Sahalia (2004), Drost, Nijman & Werker (1998).

- Future prices, from 1986 to 2007, weekly and monthly frequencies, and for various forecast horizons.
- Forecasting in real time: we use one-step-ahead out-of-sample forecasts, where parameter estimates are updated at every step of the procedure.
- In models with jumps, analytical formulae are not readily available for expected forecast errors, so we devise a simple **simulation-based** procedure to approximate these errors [Khalaf, Saphores & Bilodeau (2003); Bernard, Khalaf, Kichian & McMahon (2008)]
- Our results support the MR model over all forecast horizons considered.

$\boxed{\text{MODELS}}$

$y_t = ln(Y_t) - ln(Y_{t-1})$, $Y_t =$ nominal price level

► $\boxed{\text{GARCH-M(1,1)}}$

relationship between returns & time-varying risk, via ϕ

$$\begin{aligned} y_t &= v_t + \beta h_t, \\ v_t &= \mu + \sqrt{h_t} z_t, \\ h_t &= \alpha_0 + \alpha_1 (y_{t-1} - \mu)^2 + \phi h_{t-1} \\ & \quad z_t \overset{i.i.d.}{\sim} N(0,1). \end{aligned}$$

► $\boxed{\text{GARCH(1,1)}}$ with cond. non-normality

$\beta = 0$ &

$z_t \overset{i.i.d.}{\sim}$ student-t(τ) where τ is unknown.

$\boxed{\text{EGARCH(1,1)}}$ sign of shocks is relevant: $\beta = 0$ &

$$
\begin{aligned}
ln(h_t) \;=\; & \alpha_0 + \phi ln(h_{t-1}) + \gamma \left(\frac{y_{t-1} - \mu}{\sqrt{h_{t-1}}} \right) \\
& + \eta \left[\frac{|y_{t-1} - \mu|}{\sqrt{h_{t-1}}} - \frac{\sqrt{2}}{\sqrt{\pi}} \right].
\end{aligned}
$$

▶ $\boxed{\text{Specification with Poisson jumps:}}$

$$
y_t = v_t + \sum_{i=1}^{n_t} lnP_{it},
$$

$n_t =$ number of jumps that occur between $t-1$ & t,
$P_{it} =$ size of ith jump over this time interval.
Jumps follow a Poisson process with arrival rate λ
(a jump occurs on average, every $1/\lambda$ periods),
P_{it} i.i.d. lognormal(θ, δ^2).

$\boxed{\text{Schwartz \& Smith (2000): on commodities}}$

- MR stems from holding inventories: producers make joint decisions on production & inventory levels, accounting for a spot (sales) price & a storage price determined from marginal CY
- Two markets interact, so **equilibria** in both markets are relevant: exogenous demand shock (or changes in spot price volatility) $\rightarrow$ reactions in spot prices AND inventory adjustments $\rightarrow$ reactions in price of holding such inventory [CY] $\rightarrow$ reflected in future prices, which allows prices to revert back to trend.
- CY = counterpart of dividend yield (stock): spot price = present value of discounted future prices. Holding oil is risky $\Rightarrow$ spot prices $\neq$ expected future prices, and difference [net of storage costs] leads to marginal CY.

- Tractable time series model: MR, but a time-varying CY implies that mean to which price reverts is, itself, time-varying.

- LR (equilibrium, persistent) component$\sim$ Brownian motion, & SR (deviations, transitory) component$\sim$ Ornstein-Uhlenbeck process that reverts towards zero:

$$\begin{aligned} \ln(Y_t) &= \underbrace{\chi_t}_{SR} + \underbrace{\xi_t}_{\text{LR}} \\ \chi_t &= e^{-\kappa}\chi_{t-1} + \epsilon_t^{\chi}, \\ \xi_t &= \mu_\xi + \xi_{t-1} + \epsilon_t^{\xi} \end{aligned}$$

  $\kappa =$ rate of speed at which price reverts to its equilibrium (rate at which SR deviations disappear), $\mu_\xi =$ mean of equilibrium price, normal shocks with volatilities σ_χ & σ_ξ correlation $\rho_{\chi\xi}$.

- Note: $\chi_t = \frac{1}{\kappa}(\delta_t - \alpha)$ where $\delta_t =$ CY.

- The above specification leads to the following for future prices:

$$\ln(F_{n,t}) = e^{-kn}\chi_t + \xi_t + A(n),$$

where $F_{n,t}$ represents the market price, at time t, for a futures contract with time n until maturity

$$\xi_t = \mu_\xi + \xi_{t-1} + \epsilon_t^\xi,$$
$$\chi_t = e^{-\kappa}\chi_{t-1} + \epsilon_t^\chi,$$
$$A(n) = \left(\mu_\xi - \lambda_\xi\right) n - \left(1 - e^{-\kappa n}\right) \frac{\lambda_\chi}{\kappa}$$
$$+\frac{1}{2}\left(\left(1 - e^{-2\kappa n}\right) \frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 n + 2\left(1 - e^{-\kappa n}\right) \frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right).$$

- The system can then be written in a state-space form, estimation via Kalman filter.

Empirical Analysis

- Daily crude oil prices obtained from the U.S. Department of Energy, Energy Information Administration, for 1, 2, 3 and 4 month futures, from January 2, 1986 to January 9, 2007.

- Weekly & monthly prices: Wednesday values, & price on Wednesday ≈15th day of month.

- MR model: uses four maturities

$$MAPE = \frac{1}{K}\sum_{k=1}^{K} |ln(\hat{Y}_{T+k|T+k-1}) - ln(Y_{T+k})|,$$

$$MSPE = \frac{1}{K}\sum_{k=1}^{K} \left[ln(\hat{Y}_{T+k|T+k-1}) - ln(Y_{T+k})\right]^2.$$

- To check whether outcomes are driven by recent soaring markets, we repeat exercise, using data from January 2, 1986 to January 9, 2005 only. Qualitatively similar results

- For models with jumps:
  - We estimate parameters over sample of size T
  - Given the latter, we generate 1,000 simulated values of dependent variable $\widetilde{Y}_{T+1}$.
  - The forecast value of Y_{T+1} = average of these 1,000 $\widetilde{Y}_{T+1}$; the $T+1$ forecast error is computed.
  - The observed value of the dependent variable, Y_{T+1}, is added to the sample, the model is re-estimated, and the entire simulation process is repeated. Thus, $\widetilde{Y}_{T+2}$ is obtained, as well as the forecast error for $T+2$.
  - The above steps are repeated until $T+K$ forecast errors are obtained, which are then used to construct the MAPE and MSPE.

Horizon	1 year	3 years	5 years	
Unit root	.0115	.3558	.8256	Daily
GARCH	.0113	.3324	.8706	**1 month**
GARCH-M	.0118	.4259	1.711	
EGARCH	.0104	.3429	*.7835*	
GARCH-*t*	*.0101*	*.3318*	.7984	
GARCH + jumps	.0150	.3822	1.2272	
GARCH-*t* + jumps	.0103	.3444	.8443	
MR	**.0006**	**.0016**	**.0003**	
Unit Root	.0113	.4203	.7539	Daily
GARCH	.0102	.4083	.7601	**4 months**
GARCH-M	*.0084*	*.3909*	.9071	
EGARCH	.0105	.4200	.8004	
GARCH-T	.0096	.3998	*.7504*	
GARCH + jumps	.0126	.4635	.9787	
GARCH-*t* + jumps	.0101	.1483	.7901	
MR	**.0001**	**.0250**	**.0001**	

	1 year	3 years	5 years	
Unit Root	.0160	.2949	.7028	Weekly
GARCH	.0174	.3273	.6670	**1 month**
GARCH-M	*.0071*	*.1745*	*.4237*	
EGARCH	.0176	.3346	.6919	
GARCH-t	.5338	5.1347	10.7391	
GARCH + jumps	.0239	.4005	.8603	
GARCH-t + jumps	.0174	.3143	.6453	
MR	**.0004**	**.0003**	**.0002**	
Unit Root	.0190	.3572	.6492	Weekly
GARCH	.0196	.3827	.6183	**4 months**
GARCH-M	*.0158*	*.0996*	*.2980*	
EGARCH	.0195	.3809	.6118	
GARCH-T	.6181	6.7638	10.4958	
GARCH + jumps	.0270	.5022	.8310	
GARCH-t + jumps	.0197	.3739	.6081	
MR	**.0001**	**.0001**	**.0001**	

	1 year	3 years	5 years	
Unit Root	.0015	.2479	.8461	Monthly
GARCH	*.0014*	.2967	.8527	**1 month**
GARCH-M	.0098	*.2324*	*.7151*	
EGARCH	.0114	.3051	.8800	
GARCH-t	.0115	.3138	.8721	
GARCH + jumps	.0129	.3140	.9817	
GARCH-t + jumps	.0121	.2946	.8427	
MR	**.0004**	**.0002**	**.0002**	
Unit Root	.5363	.3196	.7431	Monthly
GARCH	.5319	.3459	.7594	**4 months**
GARCH-M	*.4961*	*.2515*	*.5483*	
EGARCH	.5320	.3453	.7556	
GARCH-t	.5319	.3445	.7593	
GARCH + jumps	.5470	.3949	.9374	
GARCH-t + jumps	.5330	.3391	.7325	
MR	**.0001**	**.0001**	**.0001**	

CONCLUSION

- Our analysis with future price data ranging from 1986 to 2007 suggests that imposing the random walk for oil prices has pronounced costs for out-of-sample forecasting.
- The mean reverting model we consider decomposes price into the sum of a Brownian motion for the long-run evolution, and an Ornstein-Uhlenbeck process for the short term deviations.
- Our results illustrate the importance of relying on a time series models for oil which fits [via the dynamics of the convenience yield] the inter-related equilibria in the spot and storage market.